COMPUTED MODAL FIELD DISTRIBUTIONS OF ISOLATED DIELECTRIC RESONATORS

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Abstract

Equivalent surface currents obtained from the solution of a surface integral equation model for a dielectric resonator are utilized to evaluate the electric and magnetic fields inside and outside of the isolated dielectric resonator. The resulting field plots permit a positive mode identification, and they are useful in designing devices to enhance coupling of or to suppress various modes.

Introduction

When the resonances in dielectric resonators are observed experimentally, it takes considerable skill to determine which frequency corresponds to a particular resonant mode. Similar difficulty in identifying the modes of dielectric resonators is encountered in numerical determination of resonant frequencies. In the latter case, for example, the complex frequencies for rotationally symmetric bodies may be obtained by finding zeroes of the matrix equation determinant with a fixed value of m (azimuthal modal number). When a natural frequency is found, there is no immediate indication of the value of the other two modal numbers n and ℓ which correspond to this particular natural frequency. In this paper we address the problem of identifying the individual modes of an isolated dielectric resonator by calculating the detailed field distribution for each of the encountered resonances.

Computational Procedure

The computational procedure for the fields is based on the surface integral equation for rotational dielectric bodies, as described in [1]. The matrix equation developed via the method of moments is partitioned in the following way:

$$\left[\begin{array}{c|c|c|c} \underset{\sim}{Z}_{tt} & \underset{\sim}{Z}_{t\phi} & \underset{\sim}{\Gamma}_{tt} & \underset{\sim}{\Gamma}_{t\phi} \\ \hline \underset{\sim}{Z}_{\phi t} & \underset{\sim}{Z}_{\phi\phi} & \underset{\sim}{\Gamma}_{\phi t} & \underset{\sim}{\Gamma}_{\phi\phi} \\ \hline -\underset{\sim}{\Gamma}_{tt} & -\underset{\sim}{\Gamma}_{t\phi} & \underset{\sim}{Y}_{tt} & \underset{\sim}{Y}_{t\phi} \\ \hline -\underset{\sim}{\Gamma}_{\phi t} & -\underset{\sim}{\Gamma}_{\phi\phi} & \underset{\sim}{Y}_{\phi t} & \underset{\sim}{Y}_{\phi\phi} \end{array}\right] \left[\begin{array}{c} |I_t> \\ \hline |J_\phi> \\ \hline |K_t> \\ \hline |M_\phi> \end{array}\right] = \left[\begin{array}{c} |E^i_t> \\ \hline |E^i_\phi> \\ \hline |H^i_t> \\ \hline |H^\phi_t> \end{array}\right] \quad (1)$$

Subscripts ϕ and t denote vector components in the azimuthal direction and in the direction along the generating curve for the body of revolution, respectively. I and K are the unknown "total" electric and magnetic surface currents, and J and M are the electric and magnetic surface current densities. Each of the partitions of the unknown column vector, say $|I_t>$, may consist of many elements.

In order to improve the numerical stability of the matrix for subsequent calculation of the natural frequencies and the modal fields, the variables appearing in (1) should be normalized so that all the elements within the column vector are expressed in the same physical units. This should be assured also for the column vector on the right-hand side of the equation, although for the natural response, the incident fields are non-existent. Such a normalization requires multiplications and divisions of certain blocks of rows and certain blocks of columns with various factors such as the intrinsic impedance of the dielectric material, the intrinsic impedance of free space, or the circumference of the resonator.

The consequence of this type of normalization has been a noticeable improvement in the conditioning of the matrix under consideration. As an example, the condition number based on the infinite norm [2] has been computed for the matrix appearing in (1) before and after the normalization indicated above. For the mode $HEM_{12\delta}$ with 27 points on the body (resulting in 102 × 102 matrix), the matrix condition number was reduced by a factor 10^6 .

The natural frequency of the particular mode is found by searching for a complex frequency value which causes the determinant of the matrix developed via the method of moments to vanish. Once the natural frequency is determined, the modal equivalent surface currents can be computed within a multiplicative constant using a Gaussian elimination procedure in which the value of one unknown current coefficient is arbitrarily chosen. The resulting electric and magnetic equivalent currents on the surface of the dielectric body can then be utilized to compute the electric and magnetic fields inside and surrounding the resonator. If one recognizes that the moment matrix developed from the surface integral equation actually represents a sum of the tangential interior and exterior scattered field at the body surface due to unit sources, however, these near fields can actually be computed with relatively little new programming. To accomplish this, we first introduce an artifical surface on which it is desired to compute the tangential fields, then compute fields on the artificial surface due to unit sources on the body surface using a slightly modified version of the moment matrix routine, and finally multiply by the vector of computed modal currents to obtain the actual tangential fields on the artificial surface. The major modification to the moment matrix routine involves the retention of only those terms which depend on the Green's function of the medium in which the field is to be evaluated. The disadvantage of the procedure is that two different artifical surfaces are required to find both the ρ and z components of the field, but it provides a simple and expedient means for computing the field distributions without development of a relatively complicated new program.

Field Distributions

Field distributions obtained by this procedure have been compared the the theoretical distribution for a dielectric rod waveguide, for which the solution is available in terms of Bessel functions [3]. As seen in Fig. 1, the agreement is quite good inside the resonator, but outside, one observes that the actual field of the isolated resonator decays more slowly than the field computed for a resonant section of the dielectric rod waveguide terminated by two parallel magnetic walls.

The computed vector field **F** (electric or magnetic field) for a particular mode is an exponentially decaying oscillation. Even if the decaying nature of the field is ignored, it is difficult to graphically represent the spatial distribution of the magnitude and the phase of **F**. Therefore, we display the instantaneous values of the vector

$$\mathrm{Re}(\mathbf{F}\, e^{j\omega_{mn\ell} t})$$

at several instants of time, like

$$\omega_{mn\ell} t = 0, \frac{\pi}{4}, \frac{\pi}{2}, \text{etc.}$$

In the above we use $\omega_{mn\ell}$ to represent only the imaginary part of the complex natural frequency of the mode (m,n,ℓ). A computer-generated graphical display is then used to show the field orientation at equidistant points as well as to provide some relative amplitude information. When the transverse field is more than 20dB below the maximum value of the field, the points are left blank.

Fig. 2 shows the electric field of the mode $TE_{01\delta}$ in the equatorial plane, at $\omega_{mn\ell}t=0$. One quarter of a period later, the magnetic field in the meridian plane takes the form shown in Fig. 3.

The remaining figures show the field distribution of the hybrid mode $HEM_{12\delta}$. Fig. 4 shows the electric field in the equatorial plane, and Fig. 5 shows the electric field in the meridian plane. Observe the saddle-shaped regions close to each face of the resonator where the electric field is weak. These are the regions to which the magnetic field lines of Fig. 6 are perpendicular (note the rotation in azimuth).

The field distributions such as those displayed here are useful in designing the coupling mechanisms for desired modes and in constructing devices for the suppression of undesired modes. Another application of the presented numerical procedure is foreseen in situations when coupling to nearby objects must be computed more accurately than is possible with a simple point-dipole field model of the dielectric resonator.

References

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[2] C. Klein and R. Mittra, "Stability of matrix equations arising in electromagnetics", IEEE Trans. Antennas Propagat., vol. AP-21, pp. 902-905, November, 1973.

[3] C. C. Johnson, Field and Wave Electrodynamics. New York: McGraw-Hill, 1965.

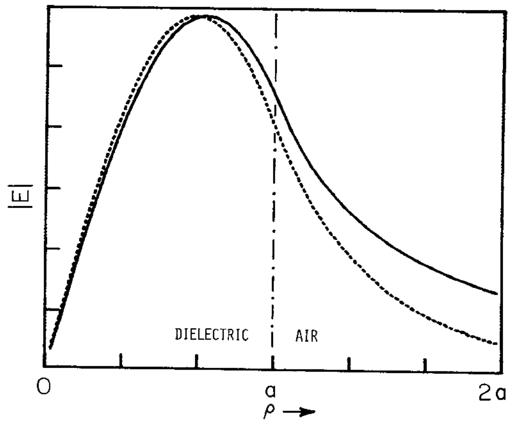

Fig. 1. Electric field vs. radial distance. Solid line: $TE_{01\delta}$ dielectric resonator. Broken line: TE_{011} dielectric rod waveguide between parallel magnetic walls. The two fields are normalized to the same maximum value.

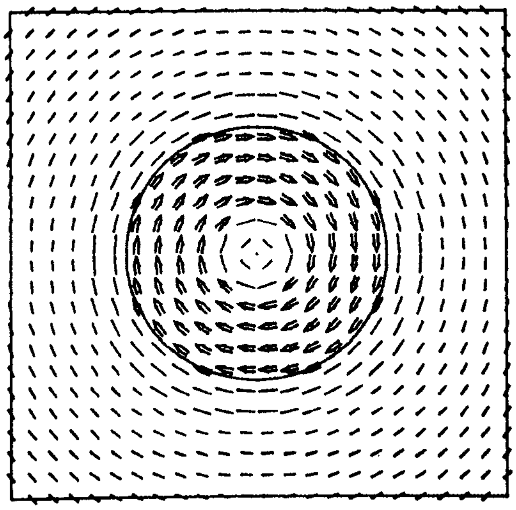

Fig. 2. $TE_{01\delta}$ mode, E-field in the equatorial plane, ωt = 0.

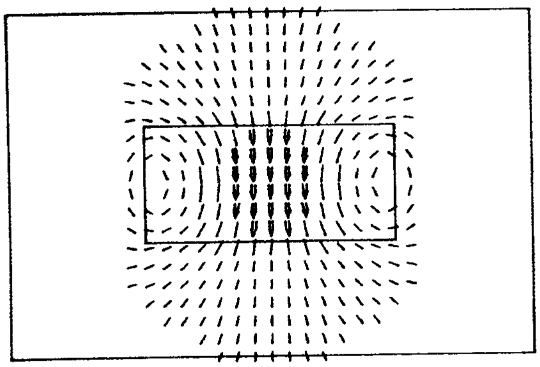

Fig. 3. $TE_{01\delta}$ mode, H-field in the meridian plane, $\omega t = \pi/2$.

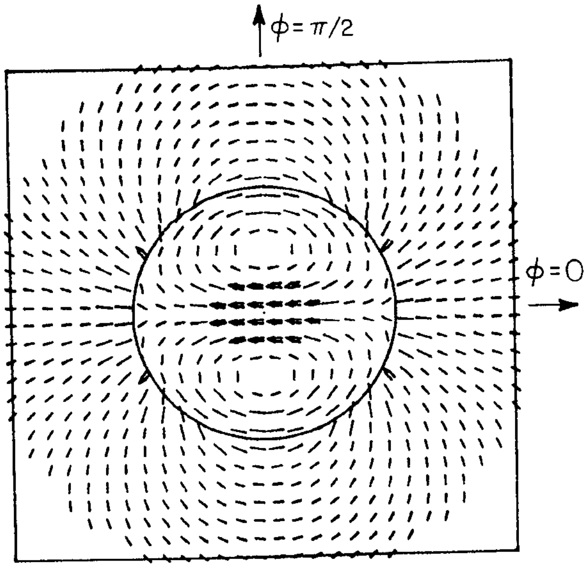

Fig. 4. $HEM_{12\delta}$ mode, E-field in the equatorial plane, ωt = 0.

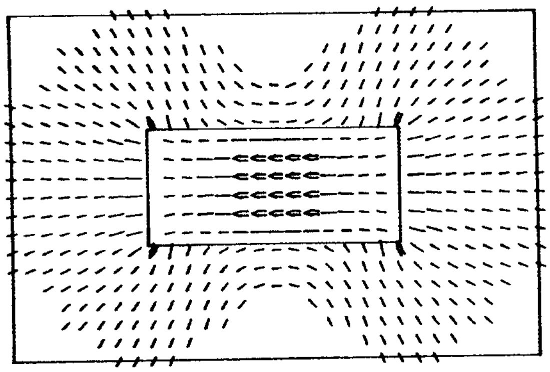

Fig. 5. $HEM_{12\delta}$ mode, E-field in the meridian plane, ϕ = 0, ωt = 0.

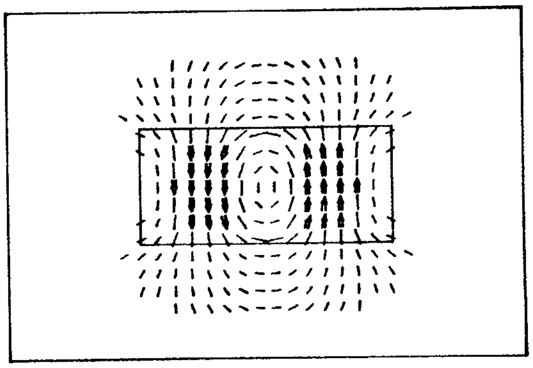

Fig. 6. $HEM_{12\delta}$ mode, H-field in the meridian plane, $\phi = \pi/2$, $\omega t = \pi/2$.